CSO$_c$ superpotentials

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Motivated by their applications to holographic RG flows and hairy black holes in Einstein-scalar systems, we present a collection of superpotentials driving the dynamics of $\mathcal{N}=2$ and $\mathcal{N}=1$ four-dimensional supergravities. These theories arise as consistent truncations of the electric/magnetic families of $\mathrm{CSO}(p,q,r)_c$ maximal supergravities, with $p+q+r=8$, discovered by Dall'Agata et al. The $\mathcal{N}=2$ and $\mathcal{N}=1$ truncations describe SU(3) and $\mathbb{Z}_2\times\mathrm{SO}(3)$ invariant sectors, respectively, and contain AdS_4 solutions preserving $\mathcal{N}=1,2,3,4$ supersymmetry within the full theories, as well as various gauge symmetries. Realisations in terms of non-geometric type IIB as well as geometric massive type IIA backgrounds are also discussed. The aim of this note is to provide easy to handle superpotentials that facilitate the study of gravitational and gauge aspects of the $\mathrm{CSO}(p,q,r)_c$ maximal supergravities avoiding the technicalities required in their construction.

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I. INTRODUCTION

The dynamics of supergravity theories in four dimensions (4D) is conjectured to describe three-dimensional field theories (at large N) in a strongly coupled regime via the gauge/gravity duality [1]. The reliability of the correspondence increases with the amount of supersymmetry, thus selecting the maximal $\mathcal{N}=8$ gauged supergravities in 4D [2–5] as preferred models where to test the duality [6–10]. However, highly supersymmetric models come along with a large number of fields filling the corresponding supermultiplets. For instance, the bosonic content of the $\mathcal{N}=8$ supergravity multiplet consists of the metric, 28 vector fields spanning a gauge group G (a.k.a. gauging) and 70 scalar fields parameterising an $\mathrm{E}_{7(7)}/\mathrm{SU}(8)$ coset space [2, 11]. Having such a large number of vectors and scalars makes the dynamics of maximal supergravities difficult to be analysed in full generality and it is at this point where the notion of consistent truncation comes to rescue.

A consistent truncation of the maximal supergravity field content implies turning off most of the fields of the theory and retaining only a small but more tractable subset of it. The truncation is called consistent if, provided a given solution to the equations of motion of the truncated theory, then those of the full theory are also satisfied. In other words, solutions of the truncated theory correspond to solutions of the full theory in which the truncated fields have been set to zero. One way of performing a consistent truncation is to keep only the subset of fields which are invariant (singlets) under the action of a compact subgroup $\mathrm{G}_0\subset\mathrm{G}$ of the gauge group G spanned by the vectors in the full theory [12]. Consistency requires the gauge group G to be a subgroup of the U-duality group $\mathrm{E}_{7(7)}$ of the maximal 4D supergravities [5, 13] which is in turn embedded into the $\mathrm{Sp}(56,\mathbb{R})$ symplectic group of electric/magnetic transformations of the theory [14]. Schematically,

$$\mathrm{G}_0\subset\mathrm{G}\subset\mathrm{E}_{7(7)}\subset\mathrm{Sp}(56,\mathbb{R})\ . \qquad (1)$$

As first noticed in the vacua classification of [15] and then made more precise in [16, 17], the embedding of the vector fields spanning G into the electromagnetic group $\mathrm{Sp}(56,\mathbb{R})$ may allow for certain freedom, or symplectic deformation. In ref. [16], a one-parameter family of $\mathrm{G}=\mathrm{SO}(8)$ gauged supergravities generalising that of de Wit and Nicolai [3, 4] was built and the structure of critical points of the associated scalar potential partially explored by looking at the $\mathrm{G}_0=\mathrm{G}_2$ consistent truncation retaining the metric and two real scalars. The electric/magnetic deformation parameter was denoted c and the family of new theories dubbed $\mathrm{SO}(8)_c$. Soon after, the scalar potential associated to different truncations of the $\mathrm{SO}(8)_c$ theories to $\mathrm{G}_0=\mathrm{SU}(3)$ [18] and $\mathrm{G}_0=\mathrm{SO}(4)$ [19] invariant sectors were put forward and their structure of critical points unraveled. In all the cases, the number of critical points, their location in field space and the corresponding value of the scalar potential V_0 turned out to change as a function of the electromagnetic or symplectic parameter c.

These remarkable facts immediately rose questions about the possible embeddings of the deformed theories (and their novel critical points) into higher-dimensional theories as well as about their conjectured gauge duals. Trying to answer these questions requires a combined understanding of: *i*) The embedding tensor formalism [5] in order to derive simple supergravity models based on electric/magnetic gaugings [17]. *ii*) Techniques of dimensional reduction/oxidation of supergravity theories

(or exceptional versions thereof [20, 21]). *iii*) Three-dimensional field theories and their connection to the theory of M2/D2-branes [6–10]. These three aspects have recently been clarified for a cousin of the $\mathrm{SO}(8)_c$ theories, namely, the electric/magnetic family of $\mathrm{ISO}(7)_c$ maximal supergravities [22, 23].

This note aims to contribute to the first "vertex of the triangle". We will present a collection of superpotentials controlling the scalar dynamics in $\mathcal{N}=2$ and $\mathcal{N}=1$ consistent truncations of $\mathrm{CSO}_c \equiv \mathrm{CSO}(p,q,r)_c$ maximal supergravities, with $p+q+r=8$, based on $\mathrm{G}_0=\mathrm{SU}(3)$ and $\mathrm{G}_0=\mathbb{Z}_2\times\mathrm{SO}(3)$ invariant sectors. Despite their simplicity, the truncated theories turn out to capture a broad set of the critical points studied in the undeformed ($c=0$) theories, their electric/magnetic deformations and more. By the end of the note, we will briefly touch on (non-)geometric string/M-theory incarnations of the CSO_c maximal supergravities.

II. ELECTRIC/MAGNETIC DEFORMATIONS

We start by introducing the families of $\mathrm{CSO}(p,q,r)_c$ maximal supergravities, with $p+q+r=8$, developed by Dall'Agata et al in [15–17]. They correspond to symplectic deformations of the $\mathrm{CSO}(p,q,r)$ theories of Hull [24–27] (see also [28]) by turning on an electric/magnetic deformation parameter c. In [17], it was shown that only the semisimple $\mathrm{SO}(p,q)\equiv\mathrm{CSO}(p,q,0)$ as well as the non-semisimple $\mathrm{ISO}(p,q)\equiv\mathrm{CSO}(p,q,1)$ gaugings turn out to admit such a deformation. Remarkably, it turns out to be a discrete (on/off) deformation for the latter [17]. The undeformed theories are then called "electric" and are recovered at $c=0$. Turning on the deformation c modifies the combinations of electric and magnetic vector fields $A_\mu^{\mathbb{M}}=(A_\mu^{[AB]},A_{\mu\,[AB]})$ which are to enter the gauge covariant derivatives D_μ of the maximal supergravity theory. We have introduced a fundamental $\mathrm{E}_{7(7)}$ index $\mathbb{M}=1,...,56$ as well as a fundamental $\mathrm{SL}(8)$ index $A=1,...,8$. Then, the $\mathrm{E}_{7(7)}\supset\mathrm{SL}(8)$ decomposition $\mathbf{56}\to\mathbf{28'}+\mathbf{28}$ simply reflects the electric (28 of them) and magnetic (28 of them) nature of the vector fields in maximal supergravity. In the undeformed case ($c=0$), only the electric vectors $A_\mu^{[AB]}$ enter the covariant derivative. After turning on the symplectic deformation, this changes to

$$\begin{aligned} D_\mu &= \partial_\mu - g\,X_{\mathbb{M}}\,A_\mu^{\mathbb{M}} \\ &= \partial_\mu - g\,\Big(X_{[AB]}\,A_\mu^{[AB]} + X^{[AB]}\,A_{\mu\,[AB]}\Big)\ , \end{aligned} \tag{2}$$

containing both electric and magnetic charges $X_{\mathbb{M}}=(X_{[AB]},X^{[AB]})$, the latter being proportional to the deformation parameter c. All the charges are specified by an *embedding tensor* ${\Theta_{\mathbb{M}}}^{\alpha}$ upon contraction with the $\mathrm{E}_{7(7)}$ generators t_α

$$X_{\mathbb{M}}={\Theta_{\mathbb{M}}}^{\alpha}\,t_\alpha \qquad \text{with} \qquad {X_{\mathbb{M}\mathbb{N}}}^{\mathbb{P}}={\Theta_{\mathbb{M}}}^{\alpha}\,{[t_\alpha]_{\mathbb{N}}}^{\mathbb{P}}\ , \tag{3}$$

where $\alpha=1,...,133$ is an adjoint index of $\mathrm{E}_{7(7)}$. Therefore, the embedding tensor ${\Theta_{\mathbb{M}}}^{\alpha}$ selects which electric/magnetic combinations of vectors are to span the gauge group $\mathrm{G}\subset\mathrm{E}_{7(7)}$ of the maximal supergravity. In order to guarantee that only 28 linearly independent combinations actually enter the gauging, a set of quadratic constraints of the form

$$\Omega^{\mathbb{M}\mathbb{N}}\,{\Theta_{\mathbb{M}}}^{\alpha}\,{\Theta_{\mathbb{N}}}^{\beta}=0 \quad \text{with} \quad \Omega^{\mathbb{M}\mathbb{N}}=\begin{pmatrix} 0_{28} & \mathbb{I}_{28} \\ -\mathbb{I}_{28} & 0_{28}\end{pmatrix}\ , \tag{4}$$

has to be satisfied by ${\Theta_{\mathbb{M}}}^{\alpha}$. In this sense, eq. (4) can be viewed as an orthogonality condition for the charges [5].

Let us first look at the family of $\mathrm{SO}(8)_c$ maximal supergravities [16] that leaves invariant the metric

$$\eta=\mathrm{diag}(1,1,1,1,1,1,1,1)\ . \tag{5}$$

In this case, the embedding tensor ${\Theta_{\mathbb{M}}}^{\alpha}$ takes the simple form

$${\Theta_{[AB]}}^{CD}=\delta_{AB}^{CD} \quad , \quad \Theta^{[AB]\,CD}=-c\,\delta_{AB}^{CD}\ . \tag{6}$$

where the index α in the ${\Theta_{\mathbb{M}}}^{\alpha}$ components in (6) runs over the linear combinations

$$T_{CD}\equiv {t_C}^{D}-{t_D}^{C}\ , \tag{7}$$

of $\mathrm{SL}(8)$ generators ${t_C}^{D}$ in the $\mathrm{SL}(8)$ decomposition of $\mathrm{E}_{7(7)}$ (see Appendix). These are precisely the 28 generators of $\mathrm{SO}(8)$. Plugging (6) into the $X_{\mathbb{M}}$ charges of (3) and substituting the result in (2) gives rise to a covariant derivative of the form

$$D_\mu=\partial_\mu-g\Big(A_\mu^{[CD]}-c\,A_{\mu\,[CD]}\Big)\ . \tag{8}$$

From (8) one sees that the parameter c sets the linear combinations of electric and magnetic vectors that enter the gauging. The electric $\mathrm{SO}(8)$ gauged supergravity of [3, 4] is recovered at $c=0$.

The $\mathrm{SO}(7,1)_c$ and $\mathrm{ISO}(7)_c\equiv\mathrm{CSO}(7,0,1)_c$ gaugings can be jointly described if applying the index splitting $A=(m,8)$ with $m=1,..,7$. The invariant metrics preserved by the gaugings can be written as

$$\eta=\mathrm{diag}(1,1,1,1,1,1,1,\epsilon_1\epsilon_2)\ , \tag{9}$$

with (ϵ_1,ϵ_2) being $(1,-1)$ and $(0,1)$ for the $\mathrm{SO}(7,1)_c$ and $\mathrm{ISO}(7)_c$ gaugings, respectively. The embedding tensor ${\Theta_{\mathbb{M}}}^{\alpha}$ is then given by

$$\begin{aligned} {\Theta_{[mn]}}^{pq}&=\delta_{mn}^{pq} \quad , \quad \Theta^{[mn]\,pq}=-\epsilon_1\,c\,\delta_{mn}^{pq}\ , \\ {\Theta_{[m8]}}^{p8}&=\delta_{m}^{p} \quad\ \ , \quad \Theta^{[m8]\,p8}=-\epsilon_2\,c\,\delta_{m}^{p}\ , \end{aligned} \tag{10}$$

where the index α in the ${\Theta_{\mathbb{M}}}^{\alpha}$ components in (10) runs over the linear combinations

$$T_{pq}\equiv {t_p}^{q}-{t_q}^{p} \qquad \text{and} \qquad T_{p8}\equiv -\epsilon_1\,{t_p}^{8}-{t_8}^{p} \tag{11}$$

of $\mathrm{SL}(8)$ generators ${t_A}^{B}$. There is then an $\mathrm{SO}(7)$ subgroup spanned by T_{pq} which is extended to either

SO(7,1) or ISO(7) by the seven generators T_{p8}. Plugging (10) into (3) and substituting again in (2) gives rise to a covariant derivative of the form

$$D_\mu = \partial_\mu - g\Big(A_\mu^{[pq]} - \epsilon_1 c\, A_{\mu\,[pq]}\Big) - g\Big(A_\mu^{[p8]} - \epsilon_2 c\, A_{\mu\,[p8]}\Big). \tag{12}$$

As noticed in [17], taking $c \neq 0$ in (12) translates into all the generators being gauged dyonically in the SO(7,1)$_c$ case whereas only the seven flat generators T_{p8} are gauged dyonically in the ISO(7)$_c$ case with $\epsilon_1 = 0$.

The SO(6,2)$_c$ and ISO(6,1)$_c \equiv$ CSO(6,1,1)$_c$ gaugings can be jointly analysed in a similar manner. This time we split the fundamental SL(8) index as $A = (1, a, 8)$ with $a = 2, ..., 7$. The invariant metrics preserved by the gaugings are now given by

$$\eta = \mathrm{diag}(-1, 1, 1, 1, 1, 1, 1, \epsilon_1\epsilon_2)\ , \tag{13}$$

with (ϵ_1, ϵ_2) being $(1, -1)$ and $(0, 1)$ for the SO(6,2)$_c$ and ISO(6,1)$_c$ gaugings, respectively. The embedding tensor $\Theta_{\mathbb{M}}{}^{\alpha}$ has components

$$\begin{array}{lll}
\Theta_{[ab]}{}^{cd} = \delta_{ab}^{cd} & , \ \Theta^{[ab]\,cd} = -\epsilon_1\, c\, \delta_{ab}^{cd} & , \\
\Theta_{[18]}{}^{18} = -1 & , \ \Theta^{[18]\,18} = -\epsilon_2\, c & , \\
\Theta_{[1b]}{}^{1d} = -\delta_b^d & , \ \Theta^{[1b]\,1d} = -\epsilon_1\, c\, \delta_b^d & , \\
\Theta_{[a8]}{}^{c8} = \delta_a^c & , \ \Theta^{[a8]\,c8} = -\epsilon_2\, c\, \delta_a^c & ,
\end{array} \tag{14}$$

with the index α in $\Theta_{\mathbb{M}}{}^{\alpha}$ running this time over the linear combinations

$$\begin{array}{ll}
T_{cd} \equiv t_c{}^d - t_d{}^c & , \ T_{18} \equiv \epsilon_1\, t_1{}^8 - t_8{}^1 \quad , \\
T_{1d} \equiv -t_1{}^d - t_d{}^1 & , \ T_{c8} \equiv -\epsilon_1\, t_c{}^8 - t_8{}^c \quad ,
\end{array} \tag{15}$$

of SL(8) generators $t_A{}^B$. The covariant derivative in this case takes the form

$$\begin{array}{rcl}
D_\mu & = & \partial_\mu \\
& - & g\Big(A_\mu^{[cd]} - \epsilon_1 c\, A_{\mu\,[cd]}\Big) + g\Big(A_\mu^{[18]} + \epsilon_2 c\, A_{\mu\,[18]}\Big) \\
& + & g\Big(A_\mu^{[1d]} + \epsilon_1 c\, A_{\mu\,[1d]}\Big) - g\Big(A_\mu^{[c8]} - \epsilon_2 c\, A_{\mu\,[c8]}\Big)\ .
\end{array} \tag{16}$$

Taking again $c \neq 0$ in (16), all the generators are gauged dyonically in the SO(6,2)$_c$ case. For the ISO(6,1)$_c$ gaugings, only the seven flat generators T_{18} and T_{c8} are gauged dyonically as $\epsilon_1 = 0$, similar to what happened in the ISO(7)$_c$ case.

The rest of CSO(p, q, r) gaugings, with $p + q + r = 8$, that admit symplectic deformations are the families of SO(5,3)$_c$ and ISO(5,2)$_c \equiv$ CSO(5,2,1)$_c$ gaugings leaving invariant the metrics

$$\eta = \mathrm{diag}(1, -1, 1, -1, 1, \epsilon_1\epsilon_2, 1, 1)\ , \tag{17}$$

with (ϵ_1, ϵ_2) being $(1, -1)$ and $(0, 1)$ respectively, as well as the SO(4,4)$_c$ and ISO(4,3)$_c \equiv$ CSO(4,3,1)$_c$ ones with invariant metrics

$$\eta = \mathrm{diag}(1, -1, 1, -1, 1, -1, 1, \epsilon_1\epsilon_2)\ , \tag{18}$$

where (ϵ_1, ϵ_2) are respectively given by $(1, -1)$ and $(0, 1)$. The derivation of the corresponding embedding tensors and covariant derivatives proceeds as for the previous cases without surprises. As before, only the seven flat generators are gauged dyonically for the ISO(5,2)$_c$ and ISO(4,3)$_c$ gaugings. For the sake of brevity, we are not presenting the expressions here.

Apart from covariantising the derivatives in (2), turning on a gauging drastically modifies the dynamics of the scalar fields in the theory by introducing a scalar potential [5]

$$V(\mathcal{M}) = \frac{g^2}{672} X_{\mathbb{MN}}{}^{\mathbb{R}} X_{\mathbb{PQ}}{}^{\mathbb{S}} \mathcal{M}^{\mathbb{MP}} \big(\mathcal{M}^{\mathbb{NQ}} \mathcal{M}_{\mathbb{RS}} + 7\, \delta_{\mathbb{R}}^{\mathbb{Q}}\, \delta_{\mathbb{S}}^{\mathbb{N}}\big)\ . \tag{19}$$

In the above formula, the 70 scalars of maximal supergravity are encoded into a coset representative $\mathcal{V} \in \mathrm{E}_{7(7)}/\mathrm{SU}(8)$ which transforms under global $\mathrm{E}_{7(7)}$ transformations from the left and local SU(8) ones from the right. This coset representative is then used to build the scalar-dependent matrix $\mathcal{M}_{\mathbb{MN}}$ as $\mathcal{M} = \mathcal{V}\,\mathcal{V}^t$, whose inverse $\mathcal{M}^{\mathbb{MN}}$ appears in (19) together with the tensor $X_{\mathbb{MN}}{}^{\mathbb{P}}$ already introduced in (3). The kinetic terms for the scalars then follow from the standard coset constructions yielding a Einstein-scalar Lagrangian of the form

$$e^{-1}\mathcal{L}_{\text{E-s}} = \tfrac{1}{2}R + \tfrac{1}{96}\mathrm{Tr}\left(D_\mu\mathcal{M}\, D^\mu\mathcal{M}^{-1}\right) - V(\mathcal{M})\ . \tag{20}$$

In this note we are setting all the vector fields to zero, so $D_\mu \to \partial_\mu$ in all the forthcoming formulas.

III. $\mathcal{N} = 2$ SUPERPOTENTIALS

After shortly reviewing the electric/magnetic CSO$_c$ gaugings of maximal supergravity, we now move on towards our actual target: provide $\mathcal{N} = 2$ truncations based on a G_0 = SU(3) invariant sector [12] that allow for an easy rewriting of the Lagrangian (20).

The SO(8)$_c$, SO(7,1)$_c$, ISO(7)$_c$, SO(6,2)$_c$ and ISO(6,1)$_c$ gaugings, they all contain an SU(3) subgroup within their maximal compact subgroups and, therefore, can accommodate such a truncation. The relevant chain of embeddings is given by

$$\mathrm{SO}(8) \supset \mathrm{SO}(7) \supset \begin{array}{c}\mathrm{SO}(6)\\ \text{or}\\ \mathrm{G}_2\end{array} \supset \mathrm{SU}(3)\ . \tag{21}$$

Truncating the $\mathcal{N} = 8$ supergravity multiplet with respect to this SU(3) preserves $\mathcal{N} = 2$ supersymmetry – the **8** gravitini of the maximal theory decompose as $\mathbf{8} \to \mathbf{1} + \mathbf{1} + \mathbf{3} + \mathbf{\bar{3}}$ under SU(3) $\subset$ SU(8), thus providing two singlets – and retains the metric, two vector fields (we are setting to zero) and six real scalars. The scalars parameterise a scalar manifold $\mathcal{M}_{\text{scal}} = \mathcal{M}_{\text{SK}} \times \mathcal{M}_{\text{QK}}$ consisting of a special Kähler (SK) piece $\mathcal{M}_{\text{SK}}$ = SU(1,1)/U(1) and a quaternionic

Kähler (QK) piece $\mathcal{M}_{\rm QK} = {\rm SU}(2,1)/{\rm U}(2)$, accounting for one vector multiplet and one hypermultiplet.

The six real scalars in the truncation are associated to SU(3)-invariant combinations of $\mathrm{E}_{7(7)}$ generators [23]. In the SL(8) basis, these are given by

$$\begin{array}{lll} g_1 &=& t_3{}^3+t_5{}^5+t_7{}^7+t_2{}^2+t_4{}^4+t_6{}^6-3\,(t_1{}^1+t_8{}^8) \;\; , \\ g_2 &=& t_8{}^1 \;\; , \\ g_3 &=& t_1{}^1-t_8{}^8 \;\; , \\ g_4 &=& t_{1238}+t_{1458}+t_{1678} \;\; , \\ g_5 &=& t_{8357}-t_{8346}-t_{8256}-t_{8247} \;\; , \\ g_6 &=& t_{8246}-t_{8257}-t_{8347}-t_{8356} \;\; , \end{array} \tag{22}$$

which are used to construct the coset representative $\mathcal{V}=\mathcal{V}_{\rm SK}\times\mathcal{V}_{\rm QK}$ upon the exponentiations

$$\begin{array}{lll} \mathcal{V}_{\rm SK} &=& e^{-12\,\chi_1\,g_4}\,e^{\frac{1}{4}\,\varphi_1\,g_1} \;\; , \\ \mathcal{V}_{\rm QK} &=& e^{a\,g_2\,-\,6\,(\zeta\,g_5\,+\,\tilde{\zeta}\,g_6)}\,e^{\phi\,g_3} \;\; . \end{array} \tag{23}$$

With the above coset representative $\mathcal{V}$, the scalar-dependent matrix $\mathcal{M}$ entering (20) is immediately obtained as $\mathcal{M}=\mathcal{V}\,\mathcal{V}^t$. Equipped with the embedding tensors of the previous section for the set of CSO_c gaugings compatible with $\mathrm{G}_0=\mathrm{SU}(3)$, it is a tedious exercise to work out the scalar Lagrangian. Plugging $\mathcal{M}$ into (20) produces kinetic terms of the form

$$\begin{array}{lll} e^{-1}\mathcal{L}_{\rm kin} &=& -\frac{3}{4}\left[(\partial\varphi_1)^2+e^{2\varphi_1}\,(\partial\chi_1)^2\right] \\ &-& (\partial\phi)^2-\frac{1}{4}e^{2\phi}\left((\partial\zeta)^2+(\partial\tilde{\zeta})^2\right) \\ &-& \frac{1}{4}e^{4\phi}\left(\partial a+\frac{1}{2}\,(\zeta\,\partial\tilde{\zeta}-\tilde{\zeta}\,\partial\zeta)\right)^2 \;\; , \end{array} \tag{24}$$

and a lengthy expression for the scalar potential V in (19) which depends on the particular choice of gauging. We will refrain from displaying the results here since, as stated in the abstract, we actually want to present them in a more concise $\mathcal{N}=2$ form.

To this end, we will start by rewriting the kinetic terms (24) encoding the geometry of the scalar manifold $\mathcal{M}_{\rm scal}$. The SK manifold spanned by the scalars (χ_1,φ_1) can be described in an $\mathcal{N}=2$ fashion by first complexifying to an upper-plane parameterisation Φ_1 and then moving to a unit-disk parameterisation z

$$(\chi_1,\varphi_1)\;\Rightarrow\;\Phi_1\equiv-\chi_1+i\,e^{-\varphi_1}\;\Rightarrow\;z\equiv\frac{\Phi_1-i}{\Phi_1+i}\;. \tag{25}$$

In this way, the kinetic terms in (24) for the scalars serving as coordinates in the SK manifold are expressed as

$$e^{-1}\mathcal{L}_{\rm kin}^{\rm SK}=3\,\frac{\partial\Phi_1\,\partial\bar{\Phi}_1}{(\Phi_1-\bar{\Phi}_1)^2}=-3\,\frac{\partial z\,\partial\bar{z}}{(1-|z|^2)^2}\;. \tag{26}$$

The geometry of the QK manifold is encoded in the kinetic terms for the four real scalars $(\phi,a,\zeta,\tilde{\zeta})$ in (24). We can alternatively describe the geometry using two real (λ,σ) and one complex ψ fields [29]

$$\lambda\equiv e^{-2\phi}\quad,\quad\sigma\equiv a\quad,\quad\psi\equiv\tfrac{1}{2}(\tilde{\zeta}+i\,\zeta)\;, \tag{27}$$

in terms of which

$$\begin{array}{lll} e^{-1}\mathcal{L}_{\rm kin}^{\rm QK} &=& -\dfrac{1}{4\lambda^2}(\partial\lambda)^2-\dfrac{1}{\lambda}(\partial\psi)(\partial\bar{\psi}) \\ &-& \dfrac{1}{4\lambda^2}\left(\partial\sigma-i\,(\psi\,\partial\bar{\psi}-\bar{\psi}\,\partial\psi)\right)^2\;, \end{array} \tag{28}$$

or using two complex fields (ζ_1,ζ_2) related to the previous ones by

$$\lambda\equiv\frac{1-|\zeta_1|^2-|\zeta_2|^2}{|1+\zeta_1|^2}\;\;,\;\;\sigma\equiv\frac{i(\zeta_1-\bar{\zeta}_1)}{|1+\zeta_1|^2}\;\;,\;\;\psi\equiv\frac{\zeta_2}{1+\zeta_1}\;, \tag{29}$$

and in terms of which the kinetic terms boil down to [29, 30]

$$\begin{array}{lll} e^{-1}\mathcal{L}_{\rm kin}^{\rm QK} &=& -\;\dfrac{(\partial\zeta_1)(\partial\bar{\zeta}_1)+(\partial\zeta_2)(\partial\bar{\zeta}_2)}{1-|\zeta_1|^2-|\zeta_2|^2} \\ &-& \dfrac{(\zeta_1\,\partial\bar{\zeta}_1+\zeta_2\,\partial\bar{\zeta}_2)(\bar{\zeta}_1\,\partial\zeta_1+\bar{\zeta}_2\,\partial\zeta_2)}{(\,1-|\zeta_1|^2-|\zeta_2|^2\,)^2}\;. \end{array} \tag{30}$$

The expressions (26), (28) and (30) often appear in the $\mathcal{N}=2$ literature (*e.g.* see [29, 30]). Here we have shown their connection to the original scalars in (23) associated to the $\mathrm{E}_{7(7)}$ generators displayed in (22).

In the spirit of [30], and when restricted to the SU(3) invariant sector of the theories, the CSO_c gaugings induce a scalar potential that can be derived from a "superpotential" $\mathcal{W}$. This superpotential takes the form [64]

$$\mathcal{W}=(1-|z|^2)^{-\frac{3}{2}}\,(1-|\zeta_{12}|^2)^{-2}\,(g\,\mathcal{W}_0+i\,gc\,\mathcal{W}_\infty)\;, \tag{31}$$

where $\mathcal{W}_0$ and $\mathcal{W}_\infty$ are functions of the complex scalar z in (25) and also of the scalars (ζ_1,ζ_2) in (29). Computing the gauge covariant derivatives $D_\mu\mathcal{M}$ for the different gaugings using (8), (12) and (16), one finds that (ζ_1,ζ_2) actually enter the (super)potential through a *gauge invariant* combination ζ_{12} satisfying $D_\mu\zeta_{12}=\partial_\mu\zeta_{12}$. For the $\mathrm{SO}(8)_c$ and $\mathrm{SO}(6{,}2)_c$ gaugings, the form of ζ_{12} is given by [30]

$$\zeta_{12}(\lambda,\sigma,|\psi|)=\frac{|\zeta_1|+i\,|\zeta_2|}{1+\sqrt{1-|\zeta_1|^2-|\zeta_2|^2}}\;. \tag{32}$$

However, only when evaluated at $\sigma=0$, the combination in (32) proves to be gauge invariant also for the $\mathrm{ISO}(7)_c$, $\mathrm{ISO}(6{,}1)_c$ and $\mathrm{SO}(7{,}1)_c$ gaugings. This is

$$\zeta_{12}\equiv\zeta_{12}(\lambda,\sigma,|\psi|)\big|_{\sigma=0}=\frac{\Phi_2-i}{\Phi_2+i}\;, \tag{33}$$

with $\Phi_2\equiv-|\psi|+i\sqrt{\lambda}$. Setting $\sigma=0$ is compatible with the extremum condition $\partial_\sigma V\big|_{\sigma=0}=0$ for all the gaugings. In fact, due to the parameterisation in

(22) and (23), the scalar σ does not enter the potential for the non-semisimple gaugings and does it quadratically (lowest order) for the semisimple ones. V is also independent of $\text{Arg}(\psi)$ for all the gaugings [65]. Fixing $\sigma = \text{Arg}(\psi) = 0$ in (28) allows for a rewriting [30]

$$e^{-1}\mathcal{L}_{\text{kin}}^{\text{QK}} = 4\,\frac{\partial\Phi_2\,\partial\bar{\Phi}_2}{(\Phi_2-\bar{\Phi}_2)^2} = -4\,\frac{\partial\zeta_{12}\,\partial\bar{\zeta}_{12}}{(1-|\zeta_{12}|^2)^2}\ , \tag{34}$$

making manifest the SK submanifold $\text{SU}(1,1)/\text{U}(1) \subset \text{SU}(2,1)/\text{U}(2)$ spanned by ζ_{12}.

The superpotential $\mathcal{W}$ in (31) is totally specified by $\mathcal{W}_0$ if $c = 0$. In this limit, superpotentials were known for the electric $\text{SO}(8)_{c=0}$ theory [30–32] as well as for the $\text{SO}(7,1)_{c=0}$, $\text{ISO}(7)_{c=0}$ and $\text{SO}(6,2)_{c=0}$ ones [31–34]. In the complementary limit $c \to \infty$, it is the function $\mathcal{W}_\infty$ that dominates. A derivation of the functions $\mathcal{W}_0$ and $\mathcal{W}_\infty$ gives the following results:

∘ *SO(8)$_c$ gaugings [18, 35]*

$$\begin{aligned}\mathcal{W}_0 &= (1+z^3)\,(1+\zeta_{12}^4) + 6\,z\,(1+z)\,\zeta_{12}^2\ ,\\ \mathcal{W}_\infty &= (1-z^3)\,(1+\zeta_{12}^4) - 6\,z\,(1-z)\,\zeta_{12}^2\ ,\end{aligned} \tag{35}$$

∘ *SO(7,1)$_c$ gaugings*

$$\begin{aligned}\mathcal{W}_0 &= \tfrac{3}{4}\,(1-\zeta_{12}^2)^2\,(1-z^2)\,(1-z)\\ &- (1+z)^3\,\zeta_{12}\,(1+\zeta_{12}^2)\ ,\\ \mathcal{W}_\infty &= \tfrac{3}{4}\,(1-\zeta_{12}^2)^2\,(1-z^2)\,(1+z)\\ &+ (1-z)^3\,\zeta_{12}\,(1+\zeta_{12}^2)\ ,\end{aligned} \tag{36}$$

∘ *ISO(7)$_c$ gaugings [23]*

$$\begin{aligned}\mathcal{W}_0 &= \tfrac{7}{8}\,(1-\zeta_{12})^4\,(1+z)^3\\ &+ 3\,(\zeta_{12}-z)\,(1+z)\,(1-\zeta_{12})^2\,(1-z\,\zeta_{12})\ ,\\ \mathcal{W}_\infty &= \tfrac{1}{8}\,(1-\zeta_{12})^4\,(1-z)^3\ ,\end{aligned} \tag{37}$$

∘ *SO(6,2)$_c$ gaugings*

$$\begin{aligned}\mathcal{W}_0 &= \tfrac{1}{2}\,(1+z)\,(1+z^2)\,(1-6\,\zeta_{12}^2+\zeta_{12}^4)\\ &- 2\,z\,(1+z)\,(1+\zeta_{12}^4)\ ,\\ \mathcal{W}_\infty &= \tfrac{1}{2}\,(1-z)\,(1+z^2)\,(1-6\,\zeta_{12}^2+\zeta_{12}^4)\\ &+ 2\,z\,(1-z)\,(1+\zeta_{12}^4)\ ,\end{aligned} \tag{38}$$

∘ *ISO(6,1)$_c$ gaugings*

$$\begin{aligned}\mathcal{W}_0 &= \tfrac{1}{8}\,(1-\zeta_{12})^2\,(1+z)\\ &\quad\big[\,(1+z^2)\big(5\,(1+\zeta_{12}^2)+14\,\zeta_{12}\big)\\ &\quad-2\,z\,\big(7\,(1+\zeta_{12}^2)+10\,\zeta_{12}\big)\big]\ ,\\ \mathcal{W}_\infty &= \tfrac{1}{8}\,(1-\zeta_{12})^4\,(1-z)^3\ .\end{aligned} \tag{39}$$

The functions $\mathcal{W}_0$ and $\mathcal{W}_\infty$ in (35)-(39) completely specify the superpotential (31) for the set of CSO_c gaugings compatible with an SU(3) truncation of maximal supergravity. For the semisimple gaugings, $\mathcal{W}_0$ and $\mathcal{W}_\infty$ are related to each other by $(z,\zeta_{12}) \leftrightarrow (-z,-\zeta_{12})$, rendering the two limits $c = 0$ and $c = \infty$ equivalent. This property no longer holds for the non-semisimple gaugings which, however, turn out to have the same $\mathcal{W}_\infty$. We will come back to these two features in the next section.

The scalar potential V is then obtained from $\mathcal{W}$ via the formula [30]

$$\begin{aligned}V &= 2\left[\frac{4}{3}\,(1-|z|^2)^2\left|\frac{\partial|\mathcal{W}|}{\partial z}\right|^2\right.\\ &\left.+\ (1-|\zeta_{12}|^2)^2\left|\frac{\partial|\mathcal{W}|}{\partial\zeta_{12}}\right|^2 - 3\,|\mathcal{W}|^2\right]\ .\end{aligned} \tag{40}$$

In addition to the superpotential $\mathcal{W}(z,\zeta_{12})$ in (31), there is a companion one $\widetilde{\mathcal{W}}(z,\zeta_{12}) = \mathcal{W}(z,\bar{\zeta}_{12})$ from which the same scalar potential in (40) follows [18, 30]. Critical points of V preserving $\mathcal{N}=2$ and $\mathcal{N}=1$ supersymmetry exist. The former satisfy $\partial|\mathcal{W}| = 0$ and $\partial|\widetilde{\mathcal{W}}| = 0$ simultaneously. In contrast, those preserving $\mathcal{N}=1$ supersymmetry satisfy either one or the other condition. For all the gaugings in (35)–(39), the potential (40) matches the one obtained from (19) using the embedding tensor formalism.

IV. $\mathcal{N}=1$ SUPERPOTENTIALS

Let us now concentrate on a different truncation based on a $\text{G}_0 = \mathbb{Z}_2 \times \text{SO}(3)$ invariant sector [36] of the maximal supergravity multiplet. The bosonic field content of this truncation consists of the metric field and six real scalars. The truncation works as follows: the $\mathbb{Z}_2$ factor truncates $\mathcal{N}=8$ supergravity to $\mathcal{N}=4$ supergravity coupled to six vector multiplets [37], whereas the additional SO(3) factor further truncates to $\mathcal{N}=1$ supergravity coupled to three chiral multiplets and no vector multiplets [38].

In addition to the gaugings of the previous section, there are $\text{SO}(5,3)_c$, $\text{SO}(4,4)_c$ and $\text{ISO}(4,3)_c$ gaugings compatible with the $\text{G}_0 = \mathbb{Z}_2 \times \text{SO}(3)$ truncation. The SO(3) is located one level lower in the chain of embeddings (21), namely,

$$\text{SO}(8) \supset \text{SO}(7) \supset \text{G}_2 \supset \begin{matrix}\text{SU}(3)\\ \text{or}\\ \text{SO}(4)\end{matrix} \supset \text{SO}(3)\ . \tag{41}$$

Under $\text{G}_0 = \mathbb{Z}_2 \times \text{SO}(3)$, the $\mathbf{8}$ gravitini of the maximal theory decompose as $\mathbf{8} \to \mathbf{1}_{(+)} + \mathbf{1}_{(-)} + \mathbf{3}_{(+)} + \mathbf{3}_{(-)}$ where the $_{(\pm)}$ subscript denotes the $\mathbb{Z}_2$-parity of the corresponding SO(3) representation. As a result, there is one $\mathbb{Z}_2$-even singlet $\mathbf{1}_{(+)}$ responsible for the $\mathcal{N}=1$ supersymmetry of the truncation. Notice that the invariant metric for

the ISO(5,2)$_c$ gaugings in (17) is simply not compatible with the above decomposition.

The six real scalars in the truncation are this time associated to the following $E_{7(7)}$ generators in the SL(8) basis [23]

$$\begin{aligned} g_1 &= \;\; t_3{}^3 + t_5{}^5 + t_7{}^7 + t_2{}^2 + t_4{}^4 + t_6{}^6 - 3\,(t_1{}^1 + t_8{}^8) \;\; , \\ g_2 &= t_1{}^1 + t_3{}^3 + t_5{}^5 + t_7{}^7 - t_2{}^2 - t_4{}^4 - t_6{}^6 - t_8{}^8 \;\; , \\ g_3 &= -t_3{}^3 - t_5{}^5 - t_7{}^7 + t_2{}^2 + t_4{}^4 + t_6{}^6 + 3\,(t_1{}^1 - t_8{}^8) \;\; , \\ g_4 &= t_{1238} + t_{1458} + t_{1678} \;\; , \\ g_5 &= t_{8246} \;\; , \\ g_6 &= t_{2578} + t_{4738} + t_{6358} \;\; , \end{aligned} \tag{42}$$

which can be used to build the coset representative $\mathcal{V} = \mathcal{V}_1 \times \mathcal{V}_2 \times \mathcal{V}_3$ upon the exponentiations

$$\begin{aligned} \mathcal{V}_1 &= e^{-12\,\chi_1\, g_4}\, e^{\frac{1}{4}\,\varphi_1\, g_1} \;\; , \\ \mathcal{V}_2 &= e^{-12\,\chi_2\, g_5}\, e^{\frac{1}{4}\,\varphi_2\, g_2} \;\; , \\ \mathcal{V}_3 &= e^{-12\,\chi_3\, g_6}\, e^{\frac{1}{4}\,\varphi_3\, g_3} \;\; . \end{aligned} \tag{43}$$

The coset representative $\mathcal{V}$ determines the scalar-dependent matrix $\mathcal{M} = \mathcal{V}\,\mathcal{V}^t$ and, after plugging into (20), one obtains the kinetic terms

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{kin}} &= -\tfrac{3}{4}\left[(\partial\varphi_1)^2 + e^{2\varphi_1}\,(\partial\chi_1)^2\right] \\ &- \tfrac{1}{4}\left[(\partial\varphi_2)^2 + e^{2\varphi_2}\,(\partial\chi_2)^2\right] \\ &- \tfrac{3}{4}\left[(\partial\varphi_3)^2 + e^{2\varphi_3}\,(\partial\chi_3)^2\right] \;\; , \end{aligned} \tag{44}$$

and again a lengthy expression for the scalar potential V that depends on the specific gauging. Analogously to the previous section, we will re-express the resulting Lagrangian in an $\mathcal{N}=1$ fashion. For this purpose, we first introduce three complex fields

$$\Phi_I = -\chi_I + i\, e^{-\varphi_I} \qquad \text{with} \qquad I = 1, 2, 3 \; , \tag{45}$$

which span a Kähler manifold $\mathcal{M}_{\text{scal}} = [\mathrm{SU}(1,1)/\mathrm{U}(1)]^3$ specified by the Kähler potential

$$K = \sum_{I=1}^{3} -n_I\, \log[-i\,(\Phi_I - \bar{\Phi}_I)] \; , \tag{46}$$

with $(n_1, n_2, n_3) = (3, 1, 3)$. In terms of (46), the kinetic terms in (44) can be rewritten as

$$e^{-1}\mathcal{L}_{\text{kin}} = -\sum_{I=1}^{3} K_{\Phi_I\bar{\Phi}_I}\, \partial\Phi_I\, \partial\bar{\Phi}_I = \sum_{I=1}^{3} n_I\, \frac{\partial\Phi_I\, \partial\bar{\Phi}_I}{(\Phi_I - \bar{\Phi}_I)^2} \; , \tag{47}$$

where $K_{\Phi_I\bar{\Phi}_I} = \partial_{\Phi_I}\partial_{\bar{\Phi}_I} K$ is the Kähler metric. The interaction between the complex scalars Φ_I is encoded into an $\mathcal{N}=1$ holomorphic superpotential W of the form

$$W = g\, W_0 + gc\, W_\infty \; , \tag{48}$$

GAUGING	p_1	p_2	p_3	p_4	q_1	q_2	q_3	q_4
SO(8)$_c$	+2	+2	+2	+2	+2	+2	+2	+2
SO(7, 1)$_c$	+2	+2	+2	−2	+2	+2	+2	−2
ISO(7)$_c$	+2	+2	+2	0	0	0	0	+2
SO(6, 2)$_c$	−2	+2	+2	−2	−2	+2	+2	−2
ISO(6, 1)$_c$	−2	+2	+2	0	0	0	0	+2
SO(5, 3)$_c$	+2	+2	−2	+2	+2	+2	−2	+2
SO(4, 4)$_c$	+2	+2	−2	−2	+2	+2	−2	−2
ISO(4, 3)$_c$	+2	+2	−2	0	0	0	0	+2

TABLE I: List of coefficients determining the W_0 and W_∞ functions in (49) for the set of CSO$_c$ gaugings compatible with $\mathrm{G}_0 = \mathbb{Z}_2 \times \mathrm{SO}(3)$.

where the functions W_0 and W_∞ are given by

$$\begin{aligned} W_0 &= p_1 \Phi_1^3 + 3\, p_2\, \Phi_1 \Phi_3^2 + 3\, p_3\, \Phi_1 \Phi_2 \Phi_3 + p_4\, \Phi_1^3 \Phi_2 \Phi_3^3 \; , \\ W_\infty &= q_1\, \Phi_2 \Phi_3^3 + 3\, q_2\, \Phi_1^2 \Phi_2 \Phi_3 + 3\, q_3\, \Phi_1^2 \Phi_3^2 + q_4 \; . \end{aligned} \tag{49}$$

The structure of monomials in W_0 and W_∞ is such that, up to an overall sign, they map into each other upon a modular transformation $\Phi_I \to -\Phi_I^{-1}$ followed by an exchange $p_i \leftrightarrow q_i$ [66]. The coefficients $p_{1,2,3,4}$ and $q_{1,2,3,4}$ in (49) are displayed in Table I for all the gaugings compatible with $\mathrm{G}_0 = \mathbb{Z}_2 \times \mathrm{SO}(3)$. Semisimple gaugings come out with $p_i = q_i \neq 0$, $\forall i = 1, ..., 4$, rendering the two limiting cases $c = 0$ and $c = \infty$ equivalent. For a generic value of c, the superpotential (48) can be thought of as an inequivalent superposition of equivalent theories. In contrast, non-semisimple gaugings have $p_i\, q_i = 0$, $\forall i = 1, ..., 4$, and this orthogonality between W_0 and W_∞ makes the $c = 0$ and $c = \infty$ cases no longer equivalent. Actually, all the non-semisimple gaugings in Table I become degenerated at the level of superpotentials in the $c \to \infty$ limit, namely,

$$\lim_{c \to \infty} W_{\mathrm{ISO}(p,q)_c} = 2\, g\, c \; . \tag{50}$$

We will recall this "universality" property later on when discussing possible higher-dimensional descriptions of the ISO$(p,q)_c$ gaugings.

Using K and W in (46) and (48), the scalar potential follows from the standard $\mathcal{N}=1$ formula

$$V = e^K \left[K^{\Phi_I\bar{\Phi}_I} (D_{\Phi_I} W)(D_{\bar{\Phi}_I} \bar{W}) - 3\, W\, \bar{W} \right] \; , \tag{51}$$

where $K^{\Phi_I\bar{\Phi}_I}$ is the inverse of the Kähler metric in (47) and $D_{\Phi_I} W = \partial_{\Phi_I} W + (\partial_{\Phi_I} K) W$ is the Kähler derivative. For all the gaugings in Table I, we have verified that the scalar potential (51) exactly reproduces the one in (19) using the embedding tensor formalism.

Taking a second look at the scalars in (22) and (42), there exists an overlapping between the $\mathcal{N}=1$ truncation

based on $\mathrm{G}_0 = \mathbb{Z}_2 \times \mathrm{SO}(3)$ and the $\mathcal{N} = 2$ truncation based on $\mathrm{G}_0 = \mathrm{SU}(3)$ discussed in the previous section. This fact was already pointed out in [35] for the case of the $\mathrm{SO}(8)_c$ gaugings and actually extends to all the CSO_c gaugings in the upper block of Table I. The overlap between the two truncations proves an $\mathcal{N} = 1$ supergravity coupled this time to two chiral multiplets and is realised by firstly identifying

$$\Phi_2 = \Phi_3 \ , \tag{52}$$

and then applying the field redefinitions in (25) and (33) to the variables z and ζ_{12}, respectively. The modular transformation $\Phi_I \rightarrow -\Phi_I^{-1}$ then translates into the $(z, \zeta_{12}) \rightarrow (-z, -\zeta_{12})$ transformation discussed in the previous section.

Next comes the $\mathrm{G}_0 = \mathrm{SO}(4) \sim \mathrm{SO}(3) \times \mathrm{SO}(3)$ invariant sector in (41). The specific embedding we consider here coincides with the one in [39] and is compatible with $\mathrm{SO}(8)_c$, $\mathrm{SO}(7{,}1)_c$, $\mathrm{ISO}(7)_c$, $\mathrm{SO}(5{,}3)_c$, $\mathrm{SO}(4{,}4)_c$ and $\mathrm{ISO}(4{,}3)_c$ gaugings. It is recovered from the $\mathrm{G}_0 = \mathbb{Z}_2 \times \mathrm{SO}(3)$ sector by identifying

$$\Phi_1 = \Phi_3 \ . \tag{53}$$

This truncation produces the gravitini decomposition $\mathbf{8} \rightarrow (\mathbf{1}, \mathbf{1}) + (\mathbf{3}, \mathbf{1}) + (\mathbf{2}, \mathbf{2})$ and also corresponds to $\mathcal{N} = 1$ supergravity coupled to two chiral multiplets [67]. For the $\mathrm{ISO}(7)_c$ gaugings, this sector has been thoroughly investigated in [23] (see [40] for the $\mathrm{SO}(8)_c$ gaugings) and found to contain the novel critical point of [39] preserving $\mathcal{N} = 3$ supersymmetry within the full theory. Here we have verified that similar $\mathcal{N} = 3$ critical points also exist for the $\mathrm{SO}(8)_c$ and the $\mathrm{SO}(7{,}1)_c$ gaugings, in agreement with [39]. In addition, the $\mathrm{SO}(7{,}1)_c$ family of gaugings was found to include an $\mathcal{N} = 4$ critical point preserving a different $\mathrm{SO}(4)$ subgroup [39]. This point corresponds to $\Phi_1 = i\,c$ and $\Phi_2 = -\bar{\Phi}_3 = e^{i\,\pi/4}$ or $-\bar{\Phi}_2 = \Phi_3 = e^{i\,\pi/4}$. In the first case, only one out of the four supersymmetries preserved by the solution lies within the $\mathcal{N} = 4$ theory obtained as $\mathcal{N} = 8 \stackrel{\mathbb{Z}_2}{\rightarrow} \mathcal{N} = 4$. In the second case, three out of the four supersymmetries belong to the $\mathcal{N} = 4$ theory.

A truncation based on $\mathrm{G}_0 = \mathrm{G}_2$ is compatible only with $\mathrm{SO}(8)_c$, $\mathrm{SO}(7{,}1)_c$ and $\mathrm{ISO}(7)_c$ gaugings [41]. The decomposition of the $\mathbf{8}$ gravitini reads $\mathbf{8} \rightarrow \mathbf{1} + \mathbf{7}$ and the truncation corresponds to $\mathcal{N} = 1$ supergravity coupled to one chiral multiplet and no vectors. This sector is recovered upon the identification

$$\Phi_1 = \Phi_2 = \Phi_3 \ , \tag{54}$$

of the three chiral fields in the $\mathrm{G}_0 = \mathbb{Z}_2 \times \mathrm{SO}(3)$ sector. In addition to the identifications in (52)–(54), there are equivalent ones obtained by discrete transformations. Finally, there also exist $\mathrm{G}_0 = \mathrm{SO}(7)$ and $\mathrm{G}_0 = \mathrm{SO}(6)$ invariant sectors but these produce non-supersymmetric truncations, so we are not considering them further in this note.

V. (NON-)GEOMETRIC STRING/M-THEORY BACKGROUNDS

Some of the $\mathcal{N} = 1$ supergravities specified in Table I have appeared in the context of non-geometric flux compactifications on toroidal backgrounds [68]. We will consider type IIB orientifolds of $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with O3/O7-planes yielding the so-called STU-models [36, 38, 42, 43] in the isotropic (or plane-exchange-symmetric) limit [44]. The connection to the dilaton (S), Kähler (T) and complex structure (U) moduli in [36, 38] is given by

$$\Phi_1 = -U^{-1} \quad , \quad \Phi_2 = S \quad , \quad \Phi_3 = T \ . \tag{55}$$

Upon a modular transformation $U \rightarrow -U^{-1}$, one obtains standard STU-models specified by the Kähler potential

$$\begin{aligned} K \ = \ & -3\,\log[-i\,(U - \bar{U})] \\ & -3\,\log[-i\,(T - \bar{T})] \ - \ \log[-i\,(S - \bar{S})] \ , \end{aligned} \tag{56}$$

and the $\mathcal{N} = 1$ superpotential (48) with

$$\begin{aligned} W_0 \ &= \ -p_1 - 3\,p_2\,U^2T^2 - 3\,p_3\,U^2ST - p_4\,ST^3 \quad , \\ W_\infty \ &= \ q_1\,U^3ST^3 + 3\,q_2\,UST + 3\,q_3\,UT^2 + q_4\,U^3 \quad . \end{aligned} \tag{57}$$

The mapping between the coefficients (p_i, q_i) in (57) and the generalised type IIB fluxes in [36, 38] reads

$$\begin{aligned} & p_1 \equiv F_3 \quad , \quad p_2 \equiv Q' \quad , \quad p_3 \equiv P \quad , \quad p_4 \equiv H_3' \quad , \\ & q_1 \equiv H_3' \quad , \quad q_2 \equiv P \quad , \quad q_3 \equiv Q' \quad , \quad q_4 \equiv F_3 \quad , \end{aligned} \tag{58}$$

where F_3 is a Ramond-Ramond three-form flux and H_3', Q' and P are (highly) non-geometric fluxes. As a result, these supergravities correspond to non-geometric type IIB toroidal backgrounds for *any* value of the electric/magnetic parameter, including the $c = 0$ case.

However, when $c = 0$, a geometric description in terms of M-theory reductions on non-compact $\mathcal{H}^{p,q,r} = \mathcal{H}^{p,q} \times T^r$ spaces, with $\mathcal{H}^{p,q}$ being a hyperboloid, is available for the $\mathrm{CSO}(p,q,r)$ gaugings [45–47]. The observation that non-geometric toroidal backgrounds may still admit geometric descriptions as non-toroidal reductions has already been made in the literature, *e.g.*, see appendix A of [48] or also [49] for a more recent discussion on non-geometric STU-models linked to compactifications of M-theory. In this regard, the Kähler potential in (56) and the W_0 superpotential in (57) provide further examples of STU-models for the electric $\mathrm{CSO}_{c=0}$ gaugings in Table I that connect to M-theory reductions on $\mathcal{H}^{p,q,r}$ spaces.

Turning on $c \neq 0$ generically causes the loss of a higher-dimensional interpretation of the CSO_c maximal supergravities. The universal limit (50) suggests a possible ten-dimensional description of the $\mathrm{ISO}(p,q)_c$ gaugings in terms of *massive* type IIA reductions on $\mathcal{H}^{p,q}$ spaces along the lines of [22, 50]. Taking $c \rightarrow \infty$, which is identified with taking a (infinitely) large Romans mass

$m = gc$ in [22], was also linked to a regular reduction of massive type IIA on T^6 in [23]. This purely magnetic limit would hide the dependence of the $\mathrm{ISO}(p,q)_c$ maximal supergravities on the $\mathcal{H}^{p,q}$ geometries clearly visible at $c = 0$, resulting in the universal superpotential of (50). For the STU-models in (57), this becomes $\lim_{c\to\infty} W_{\mathrm{ISO}(p,q)_c} = 2\,gc\,U^3$ and agrees with the identification done in [42, 44] [69] between the U^3 coupling in the superpotential and the Romans mass parameter $m = gc$ in a type IIA incarnation of the flux models [70].

Finding string/M-theory candidates to describe the semisimple $\mathrm{SO}(p,q)_c$ gaugings proves a more challenging task [71]. Symplectic deformations of semisimple gaugings – see (8), (12) and (16) – involve magnetic vectors linked to compact (com) generators of G and, consistently, two-form tensor fields in the modified electric field strengths [5, 51]. Schematically, $\mathcal{H}^{com}_{\mu\nu} = \mathcal{F}^{com}_{\mu\nu} - \frac{1}{2}\,gc\,B_{\mu\nu}$, where $\mathcal{F}^{com}_{\mu\nu}$ is the electric Yang-Mills field strength and $B_{\mu\nu}$ is a two-form field. The inclusion of tensor fields $B_{\mu\nu}$'s linked to compact generators of the U-duality group has recently been discussed in [52]. Together with those which are dual to scalars in the $\mathrm{E}_{7(7)}/\mathrm{SU}(8)$ coset, the "compact" tensor fields – dubbed "auxiliary notophs" in [52, 53] – are necessary ingredients in a superspace formulation of ungauged maximal supergravity and happen to be dual to fermion bilinears. In this sense, a gauged version of the superfield description of notophs, as well as the search for worldvolume actions, might shed light upon the microscopic origin, if any, of the $\mathrm{SO}(p,q)_c$ maximal supergravities.

VI. SUMMARY AND DISCUSSION

In this note we have provided a collection of superpotentials controlling the scalar dynamics in certain $\mathcal{N} = 2$ and $\mathcal{N} = 1$ supergravities in four dimensions. Despite their simplicity, these supergravities describe consistent truncations of the one-parameter family (with parameter c) of electric/magnetic $\mathrm{SO}(8)_c$ gauged supergravities discovered in [16] as well as its generalisation to other $\mathrm{CSO}(p,q,r)_c$ gaugings. We have studied two different truncations producing an Einstein-scalar Lagrangian of the form

$$\mathcal{L}_{\text{E-s}} = \tfrac{1}{2}\,e\,R + \mathcal{L}_{\text{kin}} - e\,V \ . \qquad (59)$$

Here is a summary of the main results:

i) The first truncation, see eq. (20), is based on a $\mathrm{G}_0 = \mathrm{SU}(3)$ invariant sector and produces an $\mathcal{N} = 2$ supergravity coupled to one vector multiplet (with complex scalar z) and one hypermultiplet (with complex scalars ζ_1 and ζ_2). $\mathcal{L}_{\text{kin}}$ consists of the two pieces (26) and (34) and, in the spirit of [30], V is obtained from the electric/magnetic superpotential $\mathcal{W}$ in (31) using (40). The different superpotentials associated to the different gaugings compatible with the truncation are listed in (35)–(39).

ii) The second truncation, see eq. (40), is based on a $\mathrm{G}_0 = \mathbb{Z}_2 \times \mathrm{SO}(3)$ invariant sector and yields an $\mathcal{N} = 1$ supergravity coupled to three chiral multiplets $\Phi_{1,2,3}$. The kinetic piece $\mathcal{L}_{\text{kin}}$ takes the form (47) and the potential V follows from the electric/magnetic superpotential W in (48) using the standard formula (51). The different superpotentials for the different gaugings compatible with the truncation are encoded in the coefficients displayed in Table I.

Applications of the CSO_c superpotentials presented in this note are immediately envisaged. The first one is the dedicated exploration and classification of critical points of the associated scalar potentials. The amount of both supersymmetric and non-supersymmetric critical points generically increases when $c \neq 0$ [15, 19, 23, 41, 54], especially for non-compact gaugings [25, 27, 33, 34, 55]. Supersymmetric extrema play a central role in the construction of BPS domain-wall solutions which are conjectured to describe RG flows via the gauge/gravity correspondence. Such BPS domain-walls have been extensively studied within the $\mathrm{G}_0 = \mathrm{SU}(3)$ invariant sector of the $\mathrm{CSO}_{c=0}$ maximal supergravities [31, 33, 34] and, more recently, also of the family of $\mathrm{SO}(8)_c$ gaugings [35, 56] using the superpotential in (31) and (35). Therefore, a second application of the CSO_c superpotentials is the systematic study of BPS domain-walls within the $\mathrm{G}_0 = \mathrm{SU}(3)$ and, even more general, within the $\mathrm{G}_0 = \mathbb{Z}_2 \times \mathrm{SO}(3)$ invariant sectors of the theories, as well as their dual RG flows. It would also be interesting to search for hairy black holes in these Einstein-scalar systems with a potential. Finally, the existence of a higher-dimensional description for the electric/magnetic families of CSO_c maximal supergravities remains one of the essential questions to be answered. By looking at the CSO_c superpotentials, the electric/magnetic deformation interpolates between two equivalent theories at $c = 0$ and $c = \infty$ for semisimple gaugings, whereas non-semisimple gaugings flow towards the universal superpotential (50) in the $c \to \infty$ limit. This fact makes the massive type IIA reductions on $\mathcal{H}^{p,q}$ spaces natural scenarios where to investigate the higher-dimensional origin of the $\mathrm{ISO}(p,q)_c$ maximal supergravities, just like the $\mathrm{ISO}(7)_c$ gaugings [23] have recently been connected to such reductions on $\mathcal{H}^{7,0} = S^6$ in [22, 50]. In contrast, the semisimple $\mathrm{SO}(p,q)_c$ gaugings remain elusive and alternative approaches, as the one based on a (gauged version of) superspace formulation of maximal supergravity [52] or those of Exceptional Generalised Geometry [57, 58] and Exceptional Field Theory [20, 21], are at this time under investigation. We hope to come back to these issues in the near future.

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Appendix: SL(8) basis of $E_{7(7)}$ generators

Let us introduce a fundamental SL(8) index $A = 1,...8$. In the SL(8) basis, the $E_{7(7)}$ generators $t_{\alpha=1,...,133}$ have a decomposition $\mathbf{133} \to \mathbf{63} + \mathbf{70}$. These are the $\mathbf{63}$ generators ${t_A}^B$ of SL(8), with ${t_A}^A = 0$, together with $\mathbf{70}$ generators $t_{ABCD} = t_{[ABCD]}$. The fundamental representation of $E_{7(7)}$ decomposes as $\mathbf{56} \to \mathbf{28} + \mathbf{28'}$, what translates into an index splitting ${}_{\mathbb{M}} \to_{[AB]} \oplus^{[AB]}$. The entries of the 56×56 matrices $[t_\alpha]_{\mathbb{M}}{}^{\mathbb{N}}$ are given by

$$\begin{aligned} [{t_A}^B]_{[CD]}{}^{[EF]} &= 4 \left(\delta^B_{[C}\, \delta^{EF}_{D]A} + \tfrac{1}{8}\, \delta^B_A\, \delta^{EF}_{CD} \right) \,, \\ [{t_A}^B]^{[EF]}{}_{[CD]} &= -[{t_A}^B]_{[CD]}{}^{[EF]} \,, \end{aligned} \tag{60}$$

for the SL(8) generators ${t_A}^B$ and by

$$\begin{aligned} [t_{ABCD}]_{[EF][GH]} &= \tfrac{2}{4!}\, \epsilon_{ABCDEFGH} \,, \\ [t_{ABCD}]^{[EF][GH]} &= 2\, \delta^{EFGH}_{ABCD} \,, \end{aligned} \tag{61}$$

for the generators t_{ABCD} extending to $E_{7(7)}$.

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[64] Notice its similarity with a central charge $Z = e^{K/2}(g\mathcal{W}_0 + igc\mathcal{W}_\infty)$ in the flux vacua attractor mechanism of [59] with K being the Kähler potential of a SK manifold $[\mathrm{SU}(1,1)/\mathrm{U}(1)] \times [\mathrm{SU}(1,1)/\mathrm{U}(1)]$ in the unit-disk parameterisation.
[65] Alternatively, it is possible to mod out the theory by a $\mathbb{Z}_2$ element [19] truncating away the (a,ζ) fields in (23) associated to the (g_2, g_5) generators in (22). This amounts to set $\sigma = \mathrm{Arg}(\psi) = 0$ in (27) and, additionally, the SU(3)-invariant vector fields are also projected out.
[66] Since the monomials in W_0 and W_∞ are related by the inversion $\Phi_I \to -\Phi_I^{-1}$, their simultaneous presence in (48) when $c \neq 0$ democratises the set of positive and negative dilaton weight-vectors in the scalar potential for semisimple gaugings. This somehow resembles the double coset construction of [11, 60].
[67] The set of CSO_c gaugings in Table I may accommodate different $\mathrm{G}_0 = \mathrm{SO}(4)$ invariant sectors. For example, some non-supersymmetric SO(4) truncations of the $\mathrm{SO}(4,4)_c$ and $\mathrm{SO}(6,2)_c$ gaugings specified by different decompositions of the **8** gravitini were investigated in [19, 54].
[68] The dependence on the duality frame (type IIA/IIB, Heterotic, M-theory,..) when it comes to classify a given supergravity as a geometric/non-geometric toroidal background has been discussed in [38, 42, 43, 61].
[69] The (S,T,U) fields here were denoted (S,U,τ) in [42] and (S,U,T) in [44].
[70] See appendix A of [23] for a detailed discussion on the non-geometric STU-model associated to the $\mathrm{ISO}(7)_c$ gaugings and its geometric origin as a massive type IIA reduction on S^6 [22, 50]. Note also that, in a IIA picture [44], the moduli T and U in (56)–(57) are identified as complex structure and Kähler moduli, respectively.
[71] For semisimple $\mathrm{SO}(p,q)_c$ gaugings one could speculate on the limiting $c = \infty$ case as an M-theory reduction, equivalent to one at $c = 0$, but on a "dual" $\mathcal{H}^{p,q}$ space. In the context of Generalised Geometry, refs [62, 63] studied the $\mathrm{SO}(8)_c$ gaugings at generic values of c and showed that these cannot be realised as a compactification of a higher-dimensional theory that is locally geometrical. This is in line with the highly non-geometric fluxes in (58) underlying the $\mathrm{SO}(p,q)_c$ gaugings.